1.) ABC Inc. has committed to making a payment of 2 million dollars in 10 years. It has purchased two assets in order to attempt to immunize this liability. It has invested \$558,394.78 in a 5-yr zero-coupon bond and \$558,394.78 in a 15-year zero-coupon bond. The annual effective yield on all assets and liabilities of ABC Inc. is 6%. Is ABC's position immunized? (Hint: use MacD and MacC)

2.) You are given the following information about a stock:

- The growth rate of dividends over the first 15 years is 2.5%
- The value of the 31^{st} dividend is \$25.00
- The growth rate of dividends after time 15 is g
- The Ratio of Div at time 21 to Div at time 7 is 1.37212
- The effective annual yield is 4%

Calculate the price of the stock.

3.) You are the Asset-Liability Manager at ManipuLife. Your company is just starting out and currently you only have 1 liability cashflow on the books. The liability amount is \$100,000 and it is to be paid exactly 3 years from now. You would like to immunize this cashflow using Macaulay Duration and Macaulay Convexity. Your boss has instructed you to **only consider the first two conditions for immunization.**

The following assets are available for purchase:

- 2-year bonds priced at par paying 10% coupons
- 10-year zero coupons bonds yielding 10%.

The current annual effective yield is 10%. How much should your company invest in each of the assets in order to immunize the liability?

4.) You have a liability where the present value of future cashflows is given by:

(continuously compounded)

PV of Liability = $\exp(r^2 - 2r)$, where r is the annual yield equal to 10%.

Calculate the ratio of the Macaulay Convexity to the ~~Modified~~ Macaulay Duration of the liability.

5.) A stock currently sells for \$137.40 and pays level dividends of \$5 every 6 months. Immediately after the dividend payment at time 7.5 it is sold for X. Determine X if the stock yields 6% annual effective over the 7.5 year period.

6.) It is January 1, 2003, and Sabrina would like to make modifications to her house. She is projecting that the following cash outlays will be required:

Modification	Date of Outlay	Amount of Outlay
Construct B-ball Court	January 1, 2004	3576
Install third bathroom	January 1, 2005	2226
Purchase Satellite Dish	July 1, 2005	2500
Add Den	January 1, 2006	5300

As building costs tend to fluctuate with interest rates, Sabrina wishes to cash flow match these anticipated expenses.

The following government bonds are available:

Maturity	Coupon Rate	Frequency of coupons
1 year	5.0%	Annual
2 years	7.0%	Annual
3 years	6.0%	Annual
4 years	10.0%	Annual

The following zero-coupon corporate bonds are available:

Maturity	Continuously Compounded Yield
6 Months	3.0%
30 Months	8.0%

Assuming all securites sell at par, determine the cost of the asset matched portfolio.

7.) You just started a new job in your companies finance department. You found some incomplete work by the previous employee. He has fit the market value (price) of an annuity to the following equation: b*exp[-ar] , where "r" represents the interest rate. He also left on his scratch pad that the duration of the annuity is 6 when r = 5%. Calculate the value of "a" in the equation.

8.) You are given the following information about Government Bonds that pay semi-annual coupons:

Maturity	Par	Coupon Rate	Yield-to-Maturity	Price
0.5 years	$ 100.00	0.00%	1.80%	$ 99.11
1.0	$ 100.00	0.00%	2.00%	$ 98.03
1.5	$ 100.00	2.50%	2.40%	$ 100.15
2.0	$ 100.00	3.00%	2.90%	$ 100.19

Calculate the spot rate at time 2 years.

9.) You are given the following information with respect to a callable bond:

Time	Expected Cashflows at a 7% annual yield
1	8.00
2	7.90
3	107.80

2.30)

Annual Yield	Bond Price
<=6.5%	104.33
7%	102.37
8%	99.76

The current yield is 7%.

Calculate the ratio of the **Effective Duration** to the **(Modified Duration** + **Macaulay Duration)** of this bond.

10.) You just walked out of your ACT2120 class all pumped up to calculate the Macaulay Duration of your own liabilities. Your records indicate the following about your liabilities:

- Car loan originally for 10,000 dollars to be repaid monthly over 5 years at 6% interest compounded monthly.
- Your car loan expires in exactly 2.5 years and you just made a payment this morning.
- Nasty bail loan from your Granny for $4000, re: that crazy night in Mexico.
- Your Granny said "I won't tell your parents, pay me when you're an FSA" and "If I were an actuary I'd say there's a 5% opportunity cost on the loan I gave you".
- You haven't written Course FM yet but you estimate that it will take you exactly 4 years from today to obtain your FSA.

What is the Macaulay Duration of your liabilities?

Term Test #4 - Solutions.

1) Condition (1) PV of assets = PV of Liabs.

$$\text{PV of liabs} = \frac{2\,000\,000}{1.06^{10}} = 1116789.60$$

$$\text{PV of assets} = 558\,394.78 + 558\,394.78 = 1\,116\,789.60$$

$\therefore$ Condition 1 is true.

Condition (2) Duration of Assets = Duration of Liabs.

$$\text{Duration of liab.} = \frac{\sum t \cdot CF_t \cdot v_i^t}{\sum CF_t \cdot v_i^t} = \frac{10 \cdot (2\,000\,000) \cdot \frac{1}{1.06^{10}}}{2\,000\,000 \cdot \frac{1}{1.06^{10}}} = 10$$

[Note to markers: score correct if....
shortcut: since only 1 liab, Duration $= t = 10$.]

Duration of Assets:

Asset 1: CF at $t=5$: $558\,394.78 \times (1.06)^5 = 747\,258.18$

$$\text{Duration} = \frac{(5)(747258.18) \cdot \frac{1}{1.06^5}}{747258.18 \cdot \frac{1}{1.06^5}} = 5$$

Asset 2: CF at $t=15$: $558.394.78 \cdot (1.06)^{15} = 1\,338\,225.59$

$$\text{Duration} = \frac{15 \cdot (1\,338\,225.59) \frac{1}{1.06^{15}}}{1\,338\,225.59 \frac{1}{1.06^{15}}} = 15$$

[Note to markers, score Durations correct if student uses shortcut again]

Term Test #4 - Solutions.

1). (continued)

Duration of Assets $= \frac{5+15}{2} = 10$ or

$$\frac{5(558\,394.78) + 15(558\,394.78)}{1116\,789.60} = 10$$

$\therefore$ condition 2 is true.

Check Condition 3: Convexity of Assets > Convexity of Liabs

Convexity of Assets:

Asset 1 : $\frac{\sum t^2 CF_t \cdot v_i^t}{\sum CF_t \cdot v_i^t} = \frac{(5)^2 558\,394.78}{558\,394.78} = 25$

Asset 2 : $15^2 = 225$ [Shortcut which is acceptable]

Convexity of Assets $= \frac{25+225}{2} = \frac{250}{2} = 125$

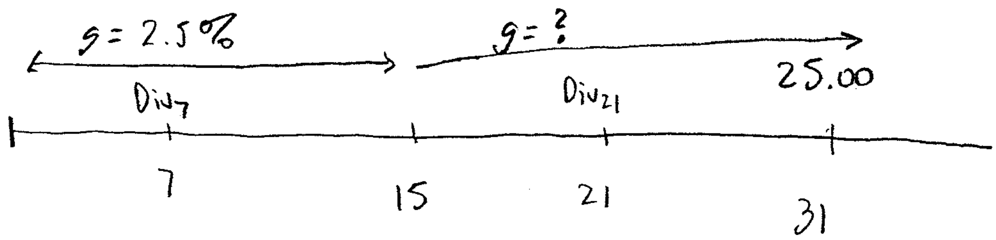
Convexity of Liab. $= 10^2 = 100$

$\therefore$ Convexity assets > Convexity of Liabs.

$\boxed{\therefore \text{ the position is immunized.}}$

Term Test #4

(2.)



$$\frac{Div_{21}}{Div_7} = 1.37212 \Rightarrow \frac{Div_I \cdot (1.025)^{14} \cdot (1+g)^6}{Div_I \cdot (1.025)^6} = 1.37212$$

$$\Rightarrow 1.025^8 \cdot (1+g)^6 = 1.37212 \qquad g = 2.0\%$$

$$Div_I = \frac{25.00}{(1.02)^{16} \cdot (1.025)^{14}} = 12.8885$$

$$Price = \frac{12.8885}{1.04} \cdot \ddot{a}_{\overline{15}|\frac{1.04}{1.025}-1} + \frac{12.8885 \cdot (1.025)^{14} \cdot (1.02)}{.04 - .02} \cdot v^{15}$$

$$Price = 12.3928 \cdot 13.576260 + \frac{18.575368}{.02} \cdot (.555265)$$

$$= 168.248 + 515.712 = \boxed{683.96}$$

(3.) $$PV \text{ of liab} = \frac{100\,000}{1.10^3} = 75131.48$$

$$MacD_L = \frac{(3)(100\,000) \cdot \frac{1}{1.10^3}}{100000 \cdot \frac{1}{1.10^3}} = 3$$

$$MacD \text{ of 2yr bonds: } \frac{(1)(10) \cdot \frac{1}{1.10} + (2)(110)\frac{1}{1.10^2}}{100} = 1.\overline{909}$$

MacD of 10 yr zero-coupon : since $t=10$, duration $=10$

Term Test #4

(3.) (Continued)

since PV_A must equal PV_L for condition 1 we must invest $x\%$ of 75131.48 in asset A, and $(1-x\%)$ of 75131.48 in Asset B, such that $Dur_A = Dur_L$:

$$Dur_L = 3 = x\%(1.90909) + (1-x\%)(10) \qquad x = 86.5168\%$$

$\boxed{\therefore \text{ invest } 86.5\% \text{ in 2yr bond and } 13.5\% \text{ in 10yr}}$

OR

$\boxed{75131.48(.86517) = 65001.39 \text{ in 2-yr and } 10{,}130.09 \text{ in } 10 \ldots}$

(4.) $MacD = -\frac{1}{P} \cdot P'$ $\qquad P' = \left[e^{r^2-2r}\right]' = (2r-2)e^{r^2-2r}$

$$\Rightarrow MacD = -\frac{(2r-2)e^{r^2-2r}}{e^{r^2-2r}} = -(2r-2) = -(2(.1)-2) = 1.8$$

$MacC = \frac{1}{P} P''$ $\qquad P'' = [P']' = \left[(2r-2)\left(e^{r^2-2r}\right)\right]' =$

$$(2)\left(e^{r^2-2r}\right) + (2r-2)(2r-2)\left(e^{r^2-2r}\right) = 2 \cdot e^{(.1)^2-2(.1)} +$$

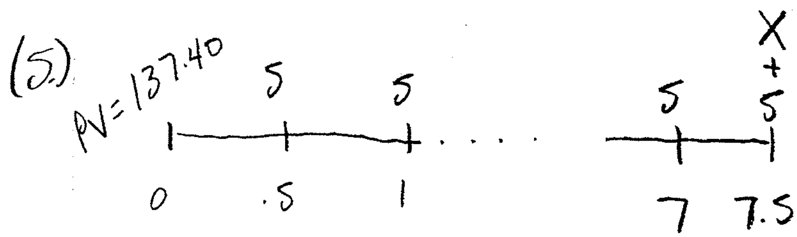
$$(2(.1)-2)^2 \cdot \left(e^{.1^2-2(.1)}\right) = 1.65392 + 1.8^2 \cdot (.826959) = 4.3333$$

$$MacC = \frac{P''}{P} \Rightarrow \frac{4.3333}{e^{.1^2-2(.1)}} = \frac{4.3333}{.826959} = 5.24$$

$$\Rightarrow \text{ratio} = \frac{5.24}{1.80} = \boxed{2.91}$$

Term Test #4

(5.)



$\frac{i^{(2)}}{2} = 1.06^{1/2} - 1 = .029563$

$$\text{Price} = 137.40 = 5 \cdot a_{\overline{15}|\frac{i^{(2)}}{2}} + X v^{7.5}$$

$$137.40 = 5 \cdot 11.97576 + X \cdot (.645961)$$

$\boxed{X = 120}$

6.)

time	1	2	2.5	3
Starting Liab. CF's	3576	2226	2500	5300
$Asset_1$ CF's	300	300	0	5300
Remaining liab. CF's	3276	1926	2500	0
$Asset_2$ CF's	0	0	2500	0
Remaining	3276	1926	0	0
$Asset_3$ CF's	126	1926	0	0
Remaining	3150	0	0	0
$Asset_4$ CF's	3150	0	0	0

$Asset_1$ face amount: $\frac{5300}{1.06} = 5000$, price = 5000 since trading at par

$Asset_2$ face amount: 2500 , price = $2500e^{-.08 \cdot 2.5} = 2046.83$

$Asset_3$ face amount: $\frac{1926}{1.07} = 1800$, price = \$1800

$Asset_4$ face amount: $\frac{3150}{1.05} = 3000$, price = 3000

$\therefore$ Total cost = $5000 + 2046.83 + 1800 + 3000 = \boxed{11846.83}$

Term Test #4

(7.) Market Value $= b \cdot e^{-ar}$ Duration $= -\frac{1}{P} \cdot P'$

$$\text{Duration} = \frac{-1}{b \cdot e^{-ar}} \cdot P' = \frac{-1}{b \cdot e^{-ar}} \cdot \left(-a \cdot b e^{-ar}\right) = \frac{a b e^{-ar}}{b \cdot e^{-ar}}$$

$$= a \quad \therefore \boxed{a = 6}$$

(8.) .5 yr bond: $s_1 = 1.80\% \Rightarrow 99.11 \approx \frac{100}{\left(1+\frac{s_1}{2}\right)}$

1-yr bond: $s_2 = 2.00\%$

1.5 yr bond: $\text{Price} = 100.15 = \frac{1.25}{\left(1+\frac{.018}{2}\right)} + \frac{1.25}{\left(1+\frac{.02}{2}\right)^2} + \frac{101.25}{\left(1+\frac{s_3}{2}\right)^3}$

$$100.15 = 1.23885 + 1.22537 + \frac{101.25}{\left(1+\frac{s_3}{2}\right)^3}$$

$$\left(1+\frac{s_3}{2}\right)^3 = \frac{101.25}{97.68578} \qquad s_3 = 2.40344\%$$

2 yr bond:

$$\text{Price} = 100.19 = \frac{1.5}{1+\frac{.018}{2}} + \frac{1.5}{\left(1+\frac{.02}{2}\right)^2} + \frac{1.5}{\left(1+\frac{.02403}{2}\right)^3} + \frac{101.5}{\left(1+\frac{s_4}{4}\right)^4}$$

$$\left(1+\frac{s_4}{2}\right)^4 = \frac{101.5}{95.79675} = 1.046933, \quad \boxed{s_4 = 2.92\%}$$

[Markers: students may use diff. notation, where $s_{.5}, s_1, s_{1.5}, s_2$ are equivalent to s_1, s_2, s_3, s_4 in my solution]

Term Test #4

(9.) $\text{EffD} = \dfrac{P_- - P_+}{P_0 \, 2(\Delta y)} = \dfrac{104.33 - 99.76}{(102.37)2(.01)} = 2.23210$

$$\text{MacD} = \frac{\sum t \cdot CF_t \cdot v_i^t}{\sum CF_t \cdot v_i^t} = \frac{(1)(8.00)\frac{1}{1.07} + (2)(7.90)\frac{1}{1.07^2} + (3)(107.80)\frac{1}{1.07^3}}{\frac{8.00}{1.07} + \frac{7.90}{1.07^2} + \frac{107.80}{1.07^3}}$$

$$= \frac{285.268}{102.373} = 2.7866 \qquad \text{ModD} = \frac{\text{MacD}}{(1.07)} = 2.6043$$

$$\therefore \text{ ratio} = \frac{2.23210}{(2.7866 + 2.6043)} = \boxed{.4141}$$

(10.) Car Loan: $10\,000 = P_{ymt} \cdot a_{\overline{60|}.005} \qquad P_{mt} = 193.328$

need Duration at $t = 2.5$ on the loan:

$$\text{PV of loan} = 193.328 \cdot a_{\overline{30|}} = 5373.37$$

$$\text{MacD} = \frac{1}{5373.37} \cdot \left[\frac{(1/12)(193.33)}{1.005^1} + \frac{(2/12)(193.33)}{1.005^2} + \dots \frac{(30/12)(193.33)}{1.005^{30}} \right] =$$

$$\frac{193.33}{5373.37} \cdot \left[P = Q = 1/12 \text{ at } .5\% \right] = \frac{193.33}{5373.37} \left[(1/12) a_{\overline{30|}} + (1/12) \left[\frac{a_{\overline{30|}} - 30 v^{30}}{.005} \right] \right]$$

$$= \frac{193.33}{5373.37} \cdot \left[\frac{27.79405}{12} + \frac{392.6324}{12} \right] = \boxed{1.2606}$$

Term Test #4

10.) (continued)

Mac. Duration of Granny's Loan $= \dfrac{(4)(4000)\frac{1}{1.05^4}}{4000 \cdot \frac{1}{1.05^4}} = 4$

MacD of Liabs $= \dfrac{5373.37 \cdot 1.2606 + \frac{4000}{1.05^4} \cdot 4}{\left(5373.37 + \frac{4000}{1.05^4}\right)} = \boxed{2.301}$